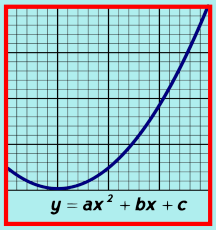
Math 25

Fall 2017

Lecture 3



A system of linear equation

$$-2\begin{cases} 2x - 3y = 4 \\ 4x - 6y = 8 \end{cases} \Rightarrow \begin{cases} -4x + 6y = -8 \\ 4x - 6y = 8 \end{cases} \Rightarrow 0 = 0$$

$$0x + 0y = 0$$

Infinitely Many Solutions

Solve

1) Graphing

2) Subs.

3) Addition / Elimination

$2x - 3y = 4$

Solve for y

$-3y = -2x + 4$

$y = \frac{2}{3}x - \frac{4}{3}$

Solution

$\left\{ \left(x, \frac{2}{3}x - \frac{4}{3} \right) \right\}$

Different x-Values, we get different y-Values

Solve

$$\begin{cases} 2x - y = 5 \\ -2x + y = -5 \end{cases}$$

$\rightarrow$ Solve for one of variable

Solve for y

$y = 2x - 5$

$\{(x, 2x-5)\}$

$0 = 0$

True

Infinitely many solutions

ex: $(1,-3)$, $(3,1)$, $(0,-5)$

Solve

$$\begin{cases} x - 2y + z = 5 \\ 2x + y - z = 7 \\ 4x - y + z = -1 \end{cases}$$

Take eqn 2 & 3, add them

$2x + y - z = 7$

$4x - y + z = -1$

$6x = 6$

$\boxed{x = 1}$

$$\begin{cases} 1 - 2y + z = 5 \\ 2 + y - z = 7 \end{cases}$$

$3 - y = 12$

$-y = 9$

$\boxed{y = -9}$

$2 - 9 - z = 7$

$-7 - z = 7$

$-z = 14$

$\boxed{z = -14}$

$x \quad y \quad z$

$(1, -9, -14)$

Solve

$$\begin{cases} 4(x-y) = 8 - z - y \\ 3 = 3x + 4z \\ -x + 3y + 3z = 1 \end{cases} \qquad \begin{cases} 4x - 4y = 8 - z - y \\ 3x + 4z = 3 \\ -x + 3y + 3z = 1 \end{cases}$$

$$\begin{cases} 4x - 3y + z = 8 \\ 3x + 4z = 3 \\ -x + 3y + 3z = 1 \end{cases}$$

Eqn 1 & Eqn 3, Add to eliminate Y

$$\begin{array}{rrrl} 4x & -3y & +z & = 8 \\ -x & +3y & +3z & = 1 \\ \hline 3x & & +4z & = 9 \end{array}$$

$$-1\begin{cases} 3x + 4z = 3 \\ 3x + 4z = 9 \end{cases}$$

$0 = 6$ False $\Rightarrow$ No Solution $\emptyset$

Solve

$$\begin{cases} -3x + 4y - z = -4 \\ x + 2y + z = 4 \\ -12x + 16y - 4z = -16 \end{cases} \Rightarrow \begin{cases} -3x + 4y - z = -4 \\ x + 2y + z = 4 \\ -3x + 4y - z = -4 \end{cases}$$

Divisible by 4

eqn 1 & 2:

$$\begin{array}{rl} -2x + 6y & = 0 \\ -x + 3y & = 0 \end{array}$$

Eqn 1 & 3:

$0 = 0 \Rightarrow$ infinitely many Solutions.

$-x + 3y = 0 \Rightarrow -x = -3y \Rightarrow x = 3y$

$(3y, y, \quad)$

Let's take eqn 2

$\boxed{x} + 2y + z = 4$

$3y + 2y + z = 4$

$5y + z = 4$

$z = 4 - 5y$

$z = 4 - 5(-2)$

$= 14$

update our general Solution

$(3y, y, 4-5y)$

ex:

$y = 0 \rightarrow (0, 0, 4)$ Soln.

$y = 1 \rightarrow (3, 1, -1)$

$y = -2 \rightarrow (-6, -2, 14)$

Recall from Algebra 1

Simplify

$$\frac{3}{x+2} - \frac{2}{x-4} = \frac{3(x-4)}{(x+2)(x-4)} - \frac{2(x+2)}{(x-4)(x+2)}$$

$$= \frac{3(x-4) - 2(x+2)}{(x+2)(x-4)}$$

$$= \frac{3x - 12 - 2x - 4}{(x+2)(x-4)} = \frac{x-16}{(x+2)(x-4)}$$

The reverse process is called Partial Fraction decomposition.

$$\frac{20x-4}{3x^2-14x-5} = \frac{20x-4}{(3x+1)(x-5)} = \frac{A}{3x+1} + \frac{B}{x-5}$$

$$3x^2-15x+x-5$$
$$= 3x^2-14x-5 \checkmark$$

$$= \frac{A(x-5)+B(3x+1)}{(3x+1)(x-5)}$$

Find Partial fraction decomposition

$$20x-4 = A(x-5)+B(3x+1)$$

Let $x=5$

$$20(5)-4 = A(5-5)+B(3\cdot 5+1)$$
$$96 = A\cdot 0 + B\cdot 16$$
$$96 = 16B \rightarrow \boxed{B=6}$$

Let $x=0$

$$20(0)-4 = A(0-5) + 6(3\cdot 0+1)$$
$$-4 = -5A+6 \qquad -10 = -5A$$
$$\boxed{A=2}$$

$$\frac{20x-4}{3x^2-14x-5} = \frac{2}{3x+1} + \frac{6}{x-5}$$

$$\frac{-10x-11}{x^2+5x-6} = \frac{-10x-11}{(x+6)(x-1)} = \frac{A}{x+6} + \frac{B}{x-1}$$

Find P.F.D.: $\frac{-7}{x+6} + \frac{-3}{x-1}$ $= \frac{A(x-1)+B(x+6)}{(x+6)(x-1)}$

$$-10x-11 = A(x-1)+B(x+6)$$

$x=1 \Rightarrow -10(1)-11 = A\cdot 0 + B\cdot 7 \qquad -21=7B$

$\boxed{B=-3}$

$x=-6 \Rightarrow -10(-6)-11 = A\cdot(-7)+B\cdot 0 \qquad 49=-7A$

$\boxed{A=-7}$

Find P.F.D:

$$\frac{13x^2+2x+45}{2x^3+18x} = \frac{13x^2+2x+45}{2x(x^2+9)} = \frac{A}{2x} + \frac{Bx+C}{x^2+9}$$

$$13x^2+2x+45 = A(x^2+9) + 2x(Bx+C)$$

$$13x^2+2x+45 = Ax^2+9A + 2Bx^2+2Cx$$

$$13x^2+2x+45 = (A+2B)x^2 + 2Cx + 9A$$

$A+2B=13 \longrightarrow 5+2B=13$

$2B=8$

$\boxed{B=4}$

$2C=2 \longrightarrow \boxed{C=1}$

$9A=45 \rightarrow \boxed{A=5}$

$$\frac{13x^2+2x+4}{2x^3+18x} = \frac{5}{2x} + \frac{4x+1}{x^2+9}$$

Find P.F.D.:

$$\frac{17x^2-7x+18}{7x^3+42x} = \frac{17x^2-7x+18}{7x(x^2+6)} = \frac{A}{7x} + \frac{Bx+C}{x^2+6}$$

$$17x^2-7x+18 = A(x^2+6) + (Bx+C)\cdot 7x$$

$$17x^2-7x+18 = Ax^2+6A + 7Bx^2+7Cx$$

$$17x^2-7x+18 = (A+7B)x^2 + 7Cx + 6A$$

$18=6A \longrightarrow \boxed{A=3}$

$-7=7C \longrightarrow \boxed{C=-1}$

$17=A+7B \longrightarrow 17=3+7B \rightarrow \boxed{B=2}$

$$\frac{3}{7x} + \frac{2x-1}{x^2+6}$$

Find P.F.D.

$$\frac{x^2+26x+100}{x^3+10x^2+25x} = \frac{x^2+26x+100}{x(x^2+10x+25)} = \frac{x^2+26x+100}{x(x+5)^2}$$

Repeated Factors

$$\frac{A(x+5)^2+Bx(x+5)+Cx}{x(x+5)^2} = \frac{A}{x} + \frac{B}{x+5} + \frac{C}{(x+5)^2}$$

$$x^2+26x+100 = A(x+5)^2 + Bx(x+5) + Cx$$

Let $x=0$ $\quad 100 = A\cdot 25 + B\cdot 0 + C\cdot 0$ $\quad \boxed{A=4}$

Let $x=-5$ $\quad -5 = A\cdot 0 + B\cdot 0 - 5C$ $\quad \boxed{C=1}$

$(-5)^2+26(-5)+100$ $\quad 127 = 4\cdot 36 + B\cdot 1\cdot 6 + 1\cdot 1$

Let $x=1$ $\quad 127 = 144 + 6B + 1$ $\quad -18 = 6B$ $\quad \boxed{B=-3}$

Find P.F.D. for

$$\frac{2x^3-11x^2-4x+24}{x^2-3x-10}$$

First Perform long division

$$\begin{array}{r} 2x \quad -5 \\ x^2-3x-10 \overline{\big) 2x^3 - 11x^2 - 4x + 24} \\ -(2x^3 - 6x^2 - 20x \quad) \\ \hline -5x^2 + 16x + 24 \\ -(-5x^2 + 15x + 50) \\ \hline x - 26 \end{array}$$

$x^2 \boxed{2x} = 2x^3$

$x^2 \boxed{-5} = -5x^2$

$$2x-5 + \frac{x-26}{x^2-3x-10}$$

Find PFD

$$\frac{x-26}{x^2-3x-10} = \frac{x-26}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$$

$$x-26 = A(x+2) + B(x-5)$$

$x=5$ $\quad 5-26 = A\cdot 7 + B\cdot 0$ $\quad -21=7A$ $\quad \boxed{A=-3}$

$x=-2$ $\quad -2-26 = A\cdot 0 + B\cdot(-7)$ $\quad -28=-7B$ $\quad \boxed{B=4}$

$$\boxed{2x-5+\frac{4}{x+2}-\frac{3}{x-5}}$$

System of non linear equations

$$\begin{cases} 2x^2+3y^2=11 \\ -2\left\{ x^2+4y^2=8 \right. \end{cases} \Rightarrow \begin{cases} 2x^2+3y^2=11 \\ -2x^2-8y^2=-16 \end{cases}$$

$$-5y^2=-5$$

$$y^2=1$$

$$y=\pm 1$$

$x^2+4y^2=8$

$x^2+4(1)^2=8$

$x^2+4=8$

$x^2=4$

$\boxed{x=\pm 2}$

$(2,1),(-2,1)$

$x^2+4y^2=8$

$x^2+4(-1)^2=8$

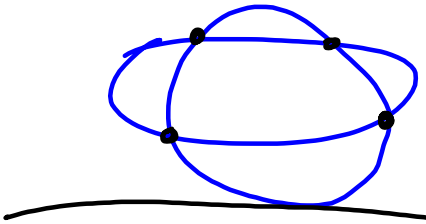
$x^2+4=8$

$x^2=4$

$\boxed{x=\pm 2}$

$(2,-1),(-2,-1)$

$$\boxed{\{(2,1),(-2,1),(-2,-1),(2,-1)\}}$$



Solve

$$\begin{cases} x^2+(y-4)^2=25 \\ x^2+y=9 \end{cases}$$

-1

$$\begin{cases} x^2+(y-4)^2=25 \\ -x^2-y=-9 \end{cases}$$

$(y-4)^2-y=16$

$(y-4)(y-4)-y=16$

$y^2-4y-4y+16-y=16$

$y^2-9y=0$

$y(y-9)=0$

$\boxed{y=0}$ or $\boxed{y=9}$

$x^2+y=9$ $\quad$ $x^2+9=9$

$x^2+0=9$ $\quad$ $x^2=0$

$x^2=9$ $\quad$ $x=0$

$x=\pm 3$ $\quad$ $\boxed{\{(3,0),(-3,0),(0,9)\}}$

Solve

$$\begin{cases} y=-x^2+6x-9 \\ y=x^2-2x-3 \end{cases}$$

$x^2-2x-3=-x^2+6x-9$

$x^2-2x-3+x^2-6x+9=0$

$2x^2-8x+6=0$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3$ $\quad$ $x=1$

when $x=1$,

$y=1^2-2(1)-3$

$\boxed{y=-4} \Rightarrow (1,-4)$

when $x=3$

$y=3^2-2(3)-3$

$=9-6-3=0 \Rightarrow (3,0)$

$\{(1,-4),(3,0)\}$

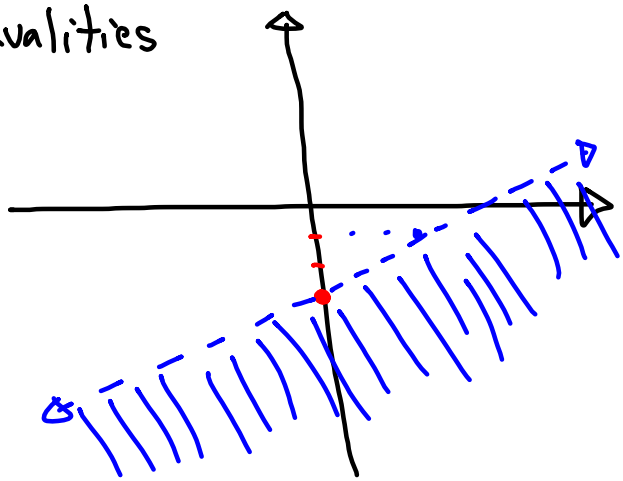
work with inequalities

Graph & shade

$y < \frac{2}{3}x - 3$

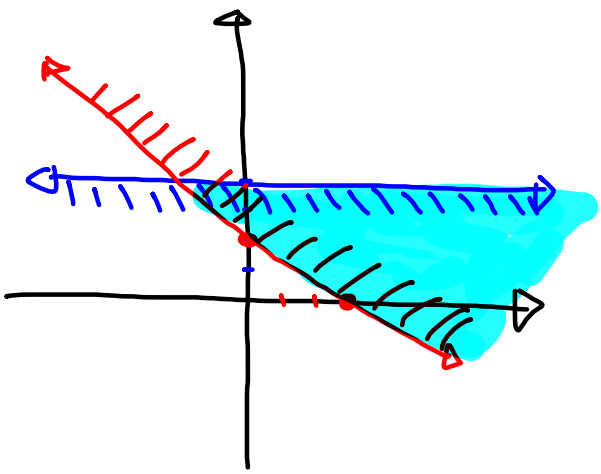
Slope-Int

Broken line



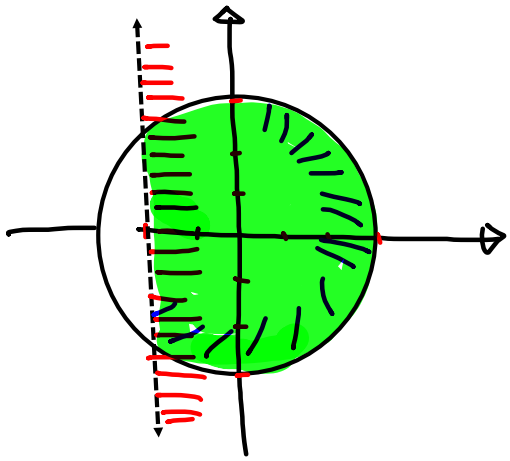
Graph & shade

$$\begin{cases} y \leq 4 \\ y \geq \frac{-2}{3}x + 2 \end{cases}$$



Graph & Shade

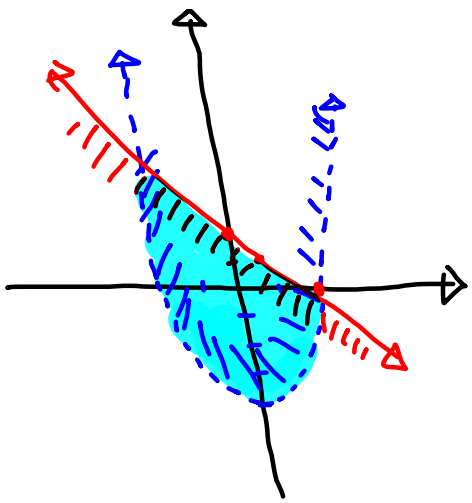
$$\begin{cases} x > -2 & \text{Vertical} \\ x^2 + y^2 \leq 9 & \text{Circle} \end{cases}$$



Graph & Shade

$$\begin{cases} y > x^2 - 4 \\ y \leq -x + 2 \end{cases}$$

x	y
0	-4
2	0
-2	0



① find P.F.D.:

$$\frac{2x^2+x-10}{x^3+5x} = \frac{2x^2+x-10}{x(x^2+5)} = \frac{A}{x} + \frac{Bx+C}{x^2+5}$$

$$2x^2+x-10 = A(x^2+5) + (Bx+C)\cdot x$$

$x=0 \Rightarrow \quad -10 = A\cdot 5 + (B\cdot 0+C)\cdot 0$ $\quad \boxed{A=-2}$

$x=1 \Rightarrow \quad -7 = -2\cdot 6 + (B\cdot 1+C)\cdot 1$ $\quad \boxed{B+C=5}$

$x=-1 \Rightarrow \quad -9 = -2\cdot 6 + (-B+C)\cdot(-1)$ $\quad \boxed{B-C=3}$

$-9 = -12 + B - C$

$2B = 8 \quad \boxed{B=4} \quad \boxed{C=1}$

Solve

$$3\begin{cases} 2x^2 - xy = 24 \\ x^2 + 3xy = -9 \end{cases} \Rightarrow \begin{cases} 6x^2 - 3xy = 72 \\ x^2 + 3xy = -9 \end{cases}$$

$7x^2 = 63$

$x^2 = 9 \qquad x = \pm 3$

when $x=3$

$3^2 + 3\cdot 3y = -9$

$9 + 9y = -9$

$9y = -18$

$\boxed{y=-2}$

$(3,-2)$

when $x=-3$

$(-3)^2 + 3(-3)y = -9$

$9 - 9y = -9$

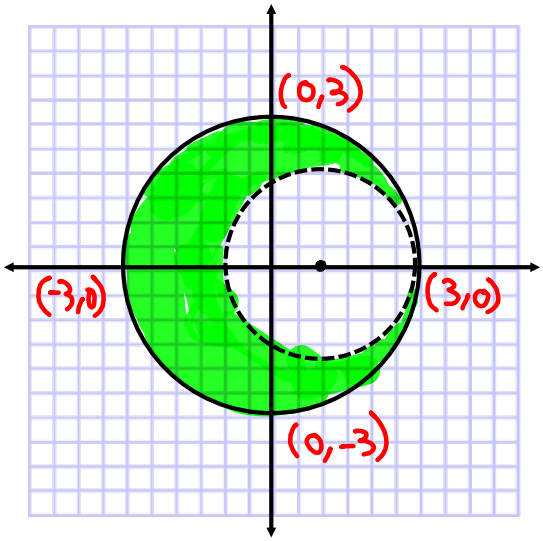
$-9y = -18$

$\boxed{y=2}$ $\quad (-3,2)$

$\{(3,-2),(-3,2)\}$

Graph & Shade

$$\begin{cases} x^2 + y^2 \le 9 \\ (x-1)^2 + y^2 > 4 \end{cases}$$



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	Sun.	10:00 AM - 3:00 PM

Looking Ahead:

Combination $\quad {}_nC_r = \dfrac{n!}{r!\cdot(n-r)!}$

$$ {}_{12}C_5 = \frac{12!}{5!\cdot 7!} = \frac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{7!}} = 11\cdot 9\cdot 8 = \boxed{792} $$

Pascal Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$(A+B)^3$

$(A+B)^4$

Find

$(A+B)^5 =$

$A^5 + A^4B + A^3B^2 + A^2B^3 + AB^4 + B^5$

${}_5C_0 = \dfrac{5!}{0!\cdot(5-0)!} = \dfrac{5!}{1\cdot 5!} = \boxed{1}$

${}_5C_1 = \dfrac{5!}{1!\cdot 4!} = \dfrac{5\cdot 4!}{1\cdot 4!} = \boxed{5}$

${}_5C_2 = \dfrac{5!}{2!\cdot 3!} = \dfrac{5\cdot 4\cdot 3!}{2\cdot 1\cdot 3!} = \boxed{10}$

${}_5C_3 \qquad {}_5C_4$

Find the first 4 terms of the expansion of $(A+B)^{12}$

$${}_{12}C_0 A^{12} + {}_{12}C_1 A^{11}B + {}_{12}C_2 A^{10}B^2 + {}_{12}C_3 A^9 B^3$$

$${}_{12}C_2 = \frac{12!}{2!\cdot(12-2)!} = \frac{12!}{2!\cdot 10!} = \frac{\cancel{12}^{6}\cdot 11\cdot\cancel{10!}}{\cancel{2}\cdot 1\cdot\cancel{10!}}$$

$$= \boxed{66}$$

Summation

End

$$\sum_{n=1}^{4}(2n+1) = \underbrace{(2\cdot 1+1)}_{n=1} + \underbrace{(2\cdot 2+1)}_{n=2} + \underbrace{(2\cdot 3+1)}_{n=3} + \underbrace{(2\cdot 4+1)}_{n=4}$$

Start

$$= 3 + 5 + 7 + 9 = \boxed{24}$$

Find

$$\sum_{n=1}^{5}(n^2-n) = \underbrace{(1^2-1)}_{n=1} + \underbrace{(2^2-2)}_{n=2} + \underbrace{(3^2-3)}_{n=3} + \underbrace{(4^2-4)}_{n=4} + \underbrace{(5^2-5)}_{n=5}$$

$$= 0 + 2 + 6 + 12 + 20 = \boxed{40}$$

Find the Sum:

$$\sum_{n=1}^{99}\left(\frac{1}{n}-\frac{1}{n+1}\right) = 1-\frac{1}{100} = \frac{100}{100}-\frac{1}{100} = \boxed{\frac{99}{100}}$$

$$= \underset{n=1}{\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)} + \underset{n=2}{\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)} + \underset{n=3}{\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)} + \cancel{\cdots} + \underset{n=99}{\left(\cancel{\frac{1}{99}}-\frac{1}{100}\right)}$$